NDA PREMIUM MOCK TEST (MATHEMATICS)

1. B B;

2. B B; $\log_{1/2}(x^2 - 6x + 12) \geq -2$ .....(i)

For log to be defined, $x^2 - 6x + 12 > 0$

$\Rightarrow (x-3)^2 + 3 > 0$, which is true $\forall x \in R$.

From (i), $x^2 - 6x + 12 \leq \left(\frac{1}{2}\right)^{-2}$

$\Rightarrow x^2 - 6x + 12 \leq 4 \Rightarrow x^2 - 6x + 8 \leq 0$

$\Rightarrow (x-2)(x-4) \leq 0 \Rightarrow 2 \leq x \leq 4$; $\therefore x \in [2, 4]$.

3. B

4. B B; $\because y = e^x$, $y = e^{-x}$ will meet, when $e^x = e^{-x}$

$\Rightarrow e^{2x} = 1$, $\therefore x = 0, y = 1$

$\therefore A$ and B meet on (0, 1), $\therefore A \cap B = \phi$.

5. A A; It is distributive law.

6. D D; It is obvious.

7. B B; $A \cup B = \{1, 2, 3, 8\}$; $A \cap B = \{3\}$

$(A \cup B) \times (A \cap B) = \{(1,3), (2,3), (3,3), (8,3)\}$.

8. A A; Let B, H, F denote the sets of members who are on the basketball team, hockey team and football team respectively.

Then we are given $n(B) = 21, n(H) = 26, n(F) = 29$

$n(H \cap B) = 14$, $n(H \cap F) = 15$, $n(F \cap B) = 12$

and $n(B \cap H \cap F) = 8$.

We have to find $n(B \cup H \cup F)$.

To find this, we use the formula

$n(B \cup H \cup F) = n(B) + n(H) + n(F)$

$-n(B \cap H) - n(H \cap F) - n(F \cap B) + n(B \cap H \cap F)$

Hence, $n(B \cup H \cup F) = (21 + 26 + 29) - (14 + 15 + 12) + 8 = 43$

Thus these are 43 members in all.

9. D B;

10. A

11. C C;

12. A

13. A A;

14. D D; Since $3 + 4i$ is a root of the equation $x^2 + px + q = 0$, therefore its other root is $3 - 4i$

Now sum of the roots $= -p$ and product of the roots $= q$

Therefore $p = -6, q = 25$.

15. B B; $x = 2 + 2^{2/3} + 2^{1/3} \Rightarrow x - 2 = 2^{2/3} + 2^{1/3}$

Cubing both sides, we get

$x^3 - 8 - 6x^2 + 12x = 6 + 6(x - 2)$

$\Rightarrow x^3 - 6x^2 + 6x = 2$.

16. D D; The given condition suggest that a lies between the roots. Let

$f(x) = 2x^2 - 2(2a+1)x + a(a+1)$

For 'a' to lie between the roots we must have Discriminant ≥ 0 and $f(a) < 0$.

Now, Discriminant ≥ 0

$\Rightarrow 4(2a+1)^2 - 8a(a+1) \geq 0$

$\Rightarrow 8(a^2 + a + 1/2) \geq 0$ which is always true.

Also $f(a) < 0 \Rightarrow 2a^2 - 2a(2a+1) + a(a+1) < 0$

$\Rightarrow -a^2 - a < 0 \Rightarrow a^2 + a > 0 \Rightarrow a(1+a) > 0$

$\Rightarrow a > 0$ or $a < -1$.

17. D

D; $S_\infty = \dfrac{a}{1-r}$ where $-1 < r < 1$ $i.e. |r| < 1$.

18. D

D; We have $\dfrac{1}{a}, \dfrac{1}{b}, \dfrac{1}{c}$ are in A.P. Let $\dfrac{1}{a} = p - q, \dfrac{1}{b} = p$ and $\dfrac{1}{c} = p + q$, where $p, q > 0$ and $p > q$. Now, substitute these values in $\dfrac{3a+2b}{2a-b} + \dfrac{3c+2b}{2c-b}$ then it reduces to $10 + \dfrac{14q^2}{p^2 - q^2}$ which is obviously greater than $10 (\text{as } p > q > 0)$.

Trick : Put $a = 1, b = \dfrac{1}{2}, c = \dfrac{1}{3}$.

The expression has the value $\dfrac{3+1}{2 - \dfrac{1}{2}} + \dfrac{1+1}{\dfrac{2}{3} - \dfrac{1}{2}} = \dfrac{8}{3} + 12 > 10$.

19. A

A; A Given series $27 + 9 + 5 . \dfrac{2}{5} + 3 . \dfrac{6}{7} + ...$

$= 27 + \dfrac{27}{3} + \dfrac{27}{5} + \dfrac{27}{7} + ... + \dfrac{27}{2n-1} + ...$

Hence n^{th} term of given series $T_n = \dfrac{27}{2n-1}$

So, $T_9 = \dfrac{27}{2 \times 9 - 1} = \dfrac{27}{17} = 1\dfrac{10}{17}$.

20. A

A; Let S_n be the sum of the given series to n terms, then

$S_n = 1 + 2x + 3x^2 + 4x^3 + ... + nx^{n-1}$ .....(i)

$xS_n = \quad x + 2x^2 + 3x^2 + ... + nx^n$ .....(ii)

Subtracting (ii) from (i), we get

$(1-x)S_n = 1 + x + x^2 + x^3 + ...\text{to} \;\; n$ terms $- nx^n$

$= \left(\dfrac{(1-x^n)}{(1-x)} \right) - nx^n$

$\Rightarrow S_n = \dfrac{(1-x^n) - nx^n(1-x)}{(1-x)^2} = \dfrac{1 - (n+1)x^n + nx^{n+1}}{(1-x)^2}$.

21. B

B; Required number of ways $= \dfrac{8!}{2!2!2!} = 5040$.

22. C

C; Either $r + 3 = 2r - 6$

or $r + 3 + 2r - 6 = 15, (\because {}^nC_r = {}^nC_{n-r})$

$\Rightarrow r = 9$ or $r = 6$.

23. C

C; Required number of ways $= {}^8C_1 + {}^8C_2 + {}^8C_3 + {}^8C_4 + {}^8C_5$

$= 8 + 28 + 56 + 70 + 56 = 218$

{Since voter may vote to one, two, three, four or all candidates}.

24. B

B; Required number of ways

$= {}^5C_3 \times {}^2C_1 \times {}^9C_7 = 10 \times 2 \times 36 = 720$.

25. C

C; Words start with D are $6! = 720$, start with E are 720, start with MD are $5! = 120$ and start with ME are 120. Now the first word starts with MO is nothing but MODESTY. Hence rank of MODESTY is 1681.

26. C

C; Required probability $= \dfrac{16}{52} = \dfrac{4}{13}$

(Since diamond has 13 cards including a king and there are another 3 kings).

27. C

C; Required probability $= {}^4C_3 . \left(\dfrac{1}{2}\right)^3 \left(\dfrac{1}{2}\right) = \dfrac{1}{4}$.

28. D

D; $4P(X=4)=P(X=2) \Rightarrow 4.{}^{6}C_4 p^4 q^2 = {}^{6}C_2 p^2 q^4$

$\Rightarrow 4p^2 = q^2 \Rightarrow 4p^2 = (1-p)^2$

$\Rightarrow 3p^2 + 2p - 1 = 0 \Rightarrow p = \frac{1}{3}$.

29. B

B; $P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

Since A and B are mutually exclusive, so

$P(A \cup B) = P(A) + P(B)$

Hence required probability $= 1 - (0.5 + 0.3) = 0.2$.

30. A

31. A

A; Mohan can gets one prize, 2 prizes or 3 prizes and his chance of failure means he get no prize.

Number of total ways $= {}^{12}C_3 = 220$

Favourable number of ways to be failure $= {}^{9}C_3 = 84$

Hence required probability $= 1 - \frac{84}{220} = \frac{34}{55}$.

32. B

B; $P(A' \cap B') = 1 - P(A \cup B)$

$= 1 - \left(\frac{1}{2} + \frac{1}{3} - \frac{7}{12}\right) = 1 - \frac{1}{4} = \frac{3}{4}$.

33. D

34. B

B; Favourable cases for one are three *i.e.* 2, 4 and 6 and for other are two *i.e.* 3, 6.

Hence required probability $= \left[\left(\frac{3 \times 2}{36}\right) 2 - \frac{1}{36}\right] = \frac{11}{36}$

{As same way happen when dice changes numbers among themselves}

35. A

A; $T_{r+1} = {}^{6}C_r x^{6-r} 3^r$

This contains x^5 if $6 - r = 5 \Rightarrow r = 1$

Coefficient of $x^5 = {}^{6}C_1 3^1 = 18$.

36. B

B; We have a = sum of the coefficient in the expansion of $(1-3x+10x^2)^n = (1-3+10)^n = (8)^n$

$\Rightarrow (1-3x+10x^2)^n = (2)^{3n}$, [Putting $x = 1$].

Now, b = sum of the coefficients in the expansion of $(1+x^2)^n = (1+1)^n = 2^n$. Clearly, $a = b^3$

37. D

D; $(1+x-2x^2)^6 = 1 + a_1 x + a_2 x^2 + ... + a_{12} x^{12}$.

Putting $x = 1$ and $x = -1$ and adding the results

$64 = 2(1+a_2+a_4+...)$

$\therefore a_2 + a_4 + a_6 + ... + a_{12} = 31$.

38. C

39. A A;

40. A A;

41. D D;

42. C C;

43. A

44. C C;

45. A

A; $y = 2x - 3 \Rightarrow x = \frac{y+3}{2}$

$\Rightarrow f^{-1}(y) = \frac{y+3}{2} \Rightarrow f^{-1}(x) = \frac{x+3}{2}$.

46. C

C; $f(x) = \log|\log x|$, $f(x)$ is defined if $|\log x| > 0$ and $x > 0$ *i.e.*,if $x > 0$ and

$x \neq 1$ $(\because |\log x| > 0 \text{ if } x \neq 1)$

$\Rightarrow x \in (0,1) \cup (1, \infty).$

47. C — C; The set B satisfied the above definition of function *f* so option C; is correct.

48. B — B; $\lim\limits_{x \to 2} \dfrac{x^n - 2^n}{x - 2} = n.2^{n-1} \Rightarrow n.2^{n-1} = 80 \Rightarrow n = 5$.

49. A — A; $\lim\limits_{x \to 2-} f(x) = 3, \lim\limits_{x \to 2+} f(x) = 3$ and $f(2) = 3$.

50. A — A; $\lim\limits_{n \to \infty} \left(\dfrac{1}{n^2} + \dfrac{2}{n^2} + \dfrac{3}{n^2} + ... + \dfrac{n}{n^2} \right)$

$$= \lim_{n \to \infty} \left(\frac{1 + 2 + 3 + ... + n}{n^2} \right) = \lim_{n \to \infty} \frac{\frac{n}{2}(n+1)}{n^2}$$

$$= \frac{1}{2} \lim_{n \to \infty} \frac{n+1}{n} = \frac{1}{2} \lim_{n \to \infty} 1 + \frac{1}{n} = \frac{1}{2}$$

51. D — D; For any $x \neq 1, 2$ we find that $f(x)$ is the quotient of two polynomials and a polynomial is everywhere continuous. Therefore $f(x)$ is continuous for all $x \neq 1, 2.$ Check continuity at $x = 1, 2.$

52. B — B; $f(x) = \left[x^2 + e^{\frac{1}{2-x}} \right]^{-1}$ and $f(2) = k$

If $f(x)$ is continuous from right at $x = 2$ then $\lim\limits_{x \to 2^+} f(x) = f(2) = k$

$$\Rightarrow \lim_{x \to 2^+} \left[x^2 + e^{\frac{1}{2-x}} \right]^{-1} = k \Rightarrow k = \lim_{h \to 0} f(2 + h)$$

$$\Rightarrow k = \lim_{h \to 0} \left[(2 + h)^2 + e^{\frac{1}{2-(2+h)}} \right]^{-1}$$

$$\Rightarrow k = \lim_{h \to 0} \left[4 + h^2 + 4h + e^{-1/h} \right]^{-1}$$

$$\Rightarrow k = [4 + 0 + 0 + e^{-\infty}]^{-1} \Rightarrow k = \frac{1}{4}.$$

53. A — A; It is formula.

54. (b) □ and □ Differentiating with respect to t, we get □ and □ (□ □ .

55. C — C; $\dfrac{d}{dx} \sin^{-1}(2ax\sqrt{1 - a^2x^2})$

Putting $ax = \sin\theta$, we get

$$= \frac{d}{dx} \sin^{-1}[2 \sin\theta\sqrt{1 - \sin^2\theta}] = \frac{d}{dx} \sin^{-1} \sin 2\theta = \frac{2a}{\sqrt{1 - a^2x^2}}$$

56. (a) □

57. B

B; Here $f(x) = |\sin 4x + 3|$

We know that minimum value of $\sin x$ is -1 and maximum is 1.

Hence minimum $|\sin 4x + 3| = |-1 + 3| = 2$ and maximum $|\sin 4x + 3| = |1 + 3| = 4$.

58. C

C; We know that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

$\Rightarrow f'(c) = \dfrac{0 - 1}{\pi / 2} = -\dfrac{2}{\pi}$ .....(i)

But $f'(x) = -\sin x \Rightarrow f'(c) = -\sin c$ ....(ii)

From (i) and (ii), we get $-\sin c = -\dfrac{2}{\pi} \Rightarrow c = \sin^{-1}\left(\dfrac{2}{\pi}\right)$.

59. C

60. C

C; $p(t) = 1000 + \dfrac{1000\,t}{100 + t^2}$

$$\frac{dp}{dt} = \frac{(100 + t^2)1000 - 1000t.2t}{(100 + t^2)^2} = \frac{1000\,(100 - t^2)}{(100 + t^2)^2}$$

For extremum, $\dfrac{dp}{dt} = 0 \Rightarrow t = 10$

Now $\left.\dfrac{dp}{dt}\right|_{t<10} > 0$ and $\left.\dfrac{dp}{dt}\right|_{t>10} < 0$

$\therefore$ At $t = 10$, $\dfrac{dp}{dt}$ change from positive to negative.

$\therefore$ p is maximum at $t = 10$.

$\therefore$ $p_{\max} = p(10) = 1000 + \dfrac{1000\,.10}{100 + 10^2} = 1050$.

61. C

C; It is a fundamental concept.

62. B

B; $\displaystyle\int \sec^{2/3} x \operatorname{cosec}^{4/3} x\, dx = \int \frac{dx}{\sin^{4/3} x \cos^{2/3} x}$

Multiplying N^r and D^r by $\cos^2 x$, we get

{Putting $\tan x = t \Rightarrow \sec^2 x\, dx = dt$}

$$= \int \frac{\sec^2 x\, dx}{\tan^{4/3} x} = \int \frac{dt}{t^{4/3}} = \frac{t^{-1/3}}{(-1/3)} + c = -3(\tan x)^{-1/3} + c.$$

63. D

D; $\displaystyle\int \frac{1}{\cos x(1 + \cos x)} dx = \int \frac{dx}{\cos x} - \int \frac{dx}{1 + \cos x}$

$$= \int \sec x\, dx - \frac{1}{2}\int \sec^2 \frac{x}{2} dx$$

$$= \log(\sec x + \tan x) - \tan \frac{x}{2} + c.$$

64. C

C; It is a fundamental property

65. C

C; $I = \displaystyle\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\cot x} + \sqrt{\tan x}} dx$ .....(i)

$$= \int_0^{\pi/2} \frac{\sqrt{\cot\left(\frac{\pi}{2} - x\right)}}{\sqrt{\cot\left(\frac{\pi}{2} - x\right)} + \sqrt{\tan\left(\frac{\pi}{2} - x\right)}} dx$$

$= \displaystyle\int_0^{\pi/2} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx$ .....(ii)

Now adding (i) and (ii), we get

$$2I=\int_0^{\pi/2}\frac{\sqrt{\cot x}+\sqrt{\tan x}}{\sqrt{\tan x}+\sqrt{\cot x}}dx=[x]_0^{\pi/2}\Rightarrow I=\frac{\pi}{4}.$$

66. D

D; Let $I=\int_0^{\pi/6}\left(2+3x^2\right)\cos 3x\,dx$

$$=\left[\frac{\sin 3x}{3}(2+3x^2)\right]_0^{\pi/6}-\int_0^{\pi/6}\frac{\sin 3x}{3}.6x.dx$$

$$=\frac{1}{36}(\pi^2+16).$$

67. A

A; Let $F_1(x)=y_1=\int_2^x(2t-5)dt$ and $F_2(x)=y_2=\int_0^x 2t\ dt$

Now point of intersection means those point at which $y_1=y_2=y\Rightarrow y_1=x^2-5x+6$ and $y_2=x^2$.

On solving, we get $x^2=x^2-5x+6\Rightarrow x=\frac{6}{5}$ and $y=x^2=\frac{36}{25}$. Thus point of intersection is $\left(\frac{6}{5},\frac{36}{25}\right)$.

68. C

C; $I=\left[\sin^{-1}\left(\frac{2x}{1+x^2}\right)\right]_0^1=\sin^{-1}(1)-\sin^{-1}(0)=\frac{\pi}{2}.$

69. B

B; Given $\sin\frac{dy}{dx}=a$; $dy=\sin^{-1}a\,dx$

Integrating both sides, $\int dy=\int\sin^{-1}a\,dx$

$y=x\sin^{-1}a+c$ and $y(0)=0+c=1$, $\therefore c=1$

$$\therefore y=x\sin^{-1}a+1\Rightarrow a=\sin\frac{y-1}{x}.$$

70. A

A; $\frac{d^2y}{dx^2}-\sqrt{\frac{dy}{dx}-3}=x\Rightarrow\frac{d^2y}{dx^2}-x=\sqrt{\frac{dy}{dx}-3}$

Squaring both sides, we get $\left(\frac{d^2y}{dx^2}-x\right)^2=\left(\frac{dy}{dx}-3\right)$

$\Rightarrow\left(\frac{d^2y}{dx^2}\right)^2+x^2-2x\frac{d^2y}{dx^2}=\frac{dy}{dx}-3$. Clearly, degree = 2.

71. C

C; Put $x+y=v$ and $1+\frac{dy}{dx}=\frac{dv}{dx}$

$$\Rightarrow\frac{dv}{dx}=v^2+1\Rightarrow\frac{dv}{v^2+1}=dx$$

On integrating, we get

$\tan^{-1}v=x+c$ or $v=\tan(x+c)\Rightarrow x+y=\tan(x+c)$

72. A

A; Given $\frac{dy}{dx}=\frac{x-y}{x+y}$. Put $y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$

$$\therefore v+x\frac{dv}{dx}=\frac{x-vx}{x+vx}$$

$$\Rightarrow v+x\frac{dv}{dx}=\frac{1-v}{1+v}\Rightarrow\frac{1+v}{2-(1+v)^2}dv=\frac{dx}{x}$$

Integrating both sides, $\int\frac{1+v}{2-(1+v)^2}dv=\int\frac{dx}{x}$

Put $(1+v)^2=t\Rightarrow 2(1+v)dv=dt$

$$\Rightarrow\frac{1}{2}\int\frac{dt}{2-t}=\int\frac{dx}{x}\Rightarrow-\frac{1}{2}\log(2-t)=\log xc$$

$$\Rightarrow-\frac{1}{2}\log[2-(1+v)^2]=\log xc$$

$$\Rightarrow-\frac{1}{2}\log[-v^2-2v+1]=\log xc$$

$\Rightarrow \log \frac{1}{\sqrt{1-2v-v^2}} = \log xc$

$\Rightarrow x^2c^2(1-2v-v^2) = 1 \Rightarrow y^2 + 2xy - x^2 = c_1$.

73. C,D

C,D; Given $\frac{d^2y}{dx^2} + x.\frac{dy}{dx} + \sin y + x^2 = 0$

The order of highest derivative = 2 and degree = 1.

74. C

C; $\tan 9^o - \tan 27^o - \tan 63^o + \tan 81^o$

$= \tan 9^o - \tan 27^o - \cot 27^o + \cot 9^o$

$= (\tan 9^o + \cot 9^o) - (\tan 27^o + \cot 27^o)$

$= \frac{\cos(9^o - 9^o)}{\sin 9^o \cos 9^o} - \frac{\cos(27^o - 27^o)}{\sin 27^o . \cos 27^o} = \frac{2}{\sin 18^o} - \frac{2}{\sin 54^o}$

$= 2\left\{\frac{\sin 54^o - \sin 18^o}{\sin 18^o \sin 54^o}\right\} = 2.\frac{2.\cos 36^o . \sin 18^o}{\sin 18^o . \sin 54^o} = 4$

75. D

D; $\cos 105° + \sin 105° = \cos(90° + 15°) + \sin(90° + 15°)$

$= \cos 15° - \sin 15° = \frac{\sqrt{3}+1}{2\sqrt{2}} - \frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$.

76. C

C; $\sin 15^o = \sin(45^o - 30^o) = \frac{\sqrt{3}-1}{2\sqrt{2}}$ = irrational

$\cos 15^o = \cos(45^o - 30^o) = \frac{\sqrt{3}+1}{2\sqrt{2}}$ =irrational

$\therefore \sin 15^o \cos 15^o = \frac{1}{2}(2\sin 15^o \cos 15^o)$

$= \frac{1}{2}\sin 30^o = \frac{1}{2}.\frac{1}{2} = \frac{1}{4}$ = rational

$\therefore \sin 15^o \cos 75^o = \sin 15^o \sin 15^o = \sin^2 15^o$

$= \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)^2 = \frac{4-2\sqrt{3}}{8}$ = irrational

77. A

A; $\sin\theta + \cos\theta = 1$

Squaring on both sides, we get

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 1$

$\therefore \sin\theta\cos\theta = 0$.

78. B

B; $\tan^2\theta = 2\tan^2\phi + 1 \Rightarrow 1 + \tan^2\theta = 2(1 + \tan^2\phi)$

$\Rightarrow \sec^2\theta = 2\sec^2\phi \Rightarrow \cos^2\phi = 2\cos^2\theta$

$\Rightarrow \cos^2\phi = 1 + \cos 2\theta \Rightarrow \sin^2\phi + \cos 2\theta = 0$.

Trick : Let $\theta = 45^o$, then $\phi = 0$

$\therefore \cos(2 \times 45^o) + \sin^2 0 = 0 + 0 = 0$.

79. B

B; Given, $\sin 2\theta + \sin 2\phi = 1/2$ …..(i)

and $\cos 2\theta + \cos 2\phi = 3/2$ …..(ii)

Square and adding ,

$\therefore (\sin^2 2\theta + \cos^2 2\theta) + (\sin^2 2\phi + \cos^2 2\phi)$

$+2[\sin 2\theta \sin 2\phi + \cos 2\theta \cos 2\phi] = 1/4 + 9/4$

$\Rightarrow \cos 2\theta \cos 2\phi + \sin 2\theta \sin 2\phi = 1/4$

$\Rightarrow \cos(2\theta - 2\phi) = 1/4 \Rightarrow \cos^2(\theta - \phi) = 5/8$.

80. (b) In

81. D

D; $\cos^{-1}\frac{1}{2}+2\sin^{-1}\frac{1}{2}=\frac{\pi}{3}+\frac{2\pi}{6}=\frac{2\pi}{3}$.

82. C

C; We have $\tan^{-1}\frac{1-x}{1+x}=\frac{1}{2}\tan^{-1}x$

$$\Rightarrow \tan^{-1}\left[\frac{1-\tan\theta}{1+\tan\theta}\right]=\frac{1}{2}\theta \quad \text{(Putting } x=\tan\theta\text{)}$$

$$\Rightarrow \tan^{-1}\left[\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}\right]=\frac{\theta}{2}$$

$$\Rightarrow \tan^{-1}\tan\left(\frac{\pi}{4}-\theta\right)=\frac{\theta}{2}\Rightarrow \frac{\pi}{4}-\theta=\frac{\theta}{2}$$

$$\Rightarrow \theta=\frac{\pi}{6}=\tan^{-1}x\Rightarrow x=\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}.$$

83. C

C; $\tan(\pi\cos\theta)=\tan\left(\frac{\pi}{2}-\pi\sin\theta\right)$

$$\therefore \sin\theta+\cos\theta=\frac{1}{2}\Rightarrow \sin\left(\theta+\frac{\pi}{4}\right)=\frac{1}{2\sqrt{2}}.$$

84. B

B; $5-5\sin^2\theta+7\sin^2\theta=6 \Rightarrow 2\sin^2\theta=1$

$$\Rightarrow \sin^2\theta=\frac{1}{2}=\sin^2\left(\frac{\pi}{4}\right)\Rightarrow \theta=n\pi\pm\frac{\pi}{4}.$$

85. C

86. B

B; Since $C=90^o$

Hence, $a=\frac{c\sin A}{\sin C}=\frac{7\sqrt{3}\sin 30^o}{\sin 90^\circ}=\frac{7\sqrt{3}}{2}$.

87. C

C; Lines $x+y=4$ and $x+y=-4$ are parallel and point (2, 2) and (–2, –2) are lies on these lines.

If point (a, a) are lie in between the lines then $a>-2$ and $a<2$ *i.e.* $-2<a<2\Rightarrow |a|<2$.

88. C

C; According to the condition $\begin{vmatrix}5 & 5 & 1\\ 10 & k & 1\\ -5 & 1 & 1\end{vmatrix}=0$

$$\Rightarrow \begin{vmatrix}5 & 5 & 1\\ 5 & k-5 & 0\\ -10 & 1-5 & 0\end{vmatrix}=0\Rightarrow k=7.$$

89. A

A; The point of intersection of $5x-6y-1=0$ and $3x+2y+5=0$ is $(-1,-1)$. Now the line perpendicular to $3x-5y+11=0$ is $5x+3y+k=0$, but it passes through $(-1,-1)\Rightarrow -5-3+k=0\Rightarrow k=8$

Hence required line is $5x+3y+8=0$.

90. (d)

Midpoint of the line joining the point □ and □ is □ i.e. (1, 2).

(In clination of straight line passing through point (–3,6) and mid point is .

(
 .

91. C

C; Let the co-ordinate of vertex A be (h,k). Then AD is perpendicular to BC, therefore $OA \perp BC$

$$\Rightarrow \frac{k-0}{h-0} \times \frac{-1}{1} = -1 \Rightarrow k = h \quad .....(i)$$

Let the coordinates of D be (α, β). Then the co-ordinates of O are $\left(\frac{2\alpha+h}{2+1}, \frac{2\beta+k}{2+1}\right)$. Therefore $\frac{2\alpha+h}{3} = 0$ and

$\frac{2\beta+k}{3} = 0 \Rightarrow \alpha = -\frac{h}{2}, \beta = \frac{-k}{2}$.

Since (α, β) lies on $x + y - 2 = 0 \Rightarrow \alpha + \beta - 2 = 0$

$\Rightarrow -h/2 - k/2 - 2 = 0 \Rightarrow h + k + 4 = 0$

$\Rightarrow 2h + 4 = 0 \Rightarrow h = k = -2$, [from (i)]

Hence the coordinates of vertex A are $(-2,-2)$.

92. C

C; From figure,

$$\left(\frac{b/2}{a/2}\right)\left(\frac{b}{-a/2}\right) = -1 \Rightarrow a^2 = 2b^2 \Rightarrow a = \pm\sqrt{2}b$$

93. D

D;

The equation of lines in intercept form are $\frac{x}{-8/a} + \frac{y}{-8/b} = 1$ .....(i)

$\frac{x}{-3} + \frac{y}{2} = 1$ .....(ii)

According to the condition, $-\frac{8}{a} = -(-3)$

$\Rightarrow a = -\frac{8}{3}$ and $-\frac{8}{b} = -(2) \Rightarrow b = 4$.

94. C

C; The four vertices on solving are $A(-3,3)$, $B(1,1)$, $C(1,-1)$ and $D(-2,-2)$. m_1 = slope of $AC = -1$, m_2 = slope of $BD = 1$; $\therefore m_1 m_2 = -1$.

Hence the angle between diagonals AC and BD is $90°$.

95. B

B; Line perpendicular to $y = mx + c$ is $y = -\frac{1}{m}x + \lambda$ and

$m\lambda = \pm a\sqrt{1+m^2}$

Hence required tangent is $my + x = \pm a\sqrt{1+m^2}$.

96. C

C; Centre is (2, 3). One end is (3, 4).

P_2 divides the join of P_1 and O in ratio of 2 : 1.

Hence P_2 is $\left(\frac{4-3}{2-1}, \frac{6-4}{2-1}\right) \equiv (1, 2)$.

97. B

98. D D; As we know, $t_1 \times t_2 = 2 \Rightarrow 2at_1 \times 2at_2 = 8a^2$.

99. C C; In the first case, eccentricity $e = \sqrt{1-(25/169)}$

In the second case, $e' = \sqrt{1-(b^2/a^2)}$

According to the given condition,

$\sqrt{1-b^2/a^2} = \sqrt{1-(25/169)}$

$\Rightarrow b/a = 5/13$, $(\because a>0, b>0)$

$\Rightarrow a/b = 13/5$.

100. A A; $|\mathbf{a}+\mathbf{b}| > |\mathbf{a}-\mathbf{b}|$

Squaring both sides, we get

$a^2 + b^2 + 2\mathbf{a}.\mathbf{b} > a^2 + b^2 - 2\mathbf{a}.\mathbf{b}$

$\Rightarrow 4\mathbf{a}.\mathbf{b} > 0 \Rightarrow \cos\theta > 0$. Hence $\theta < 90°$, (acute).

101. D D; We know that P will be the mid point of AC and BD

$\therefore \quad \overrightarrow{OA} + \overrightarrow{OC} = 2\overrightarrow{OP}$ ......(i)

and $\overrightarrow{OB} + \overrightarrow{OD} = 2\overrightarrow{OP}$ .....(ii)

Adding (i) and (ii), we get, $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} = 4\overrightarrow{OP}$.

102. B B; $|\mathbf{a} \times \mathbf{i}|^2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ 1 & 0 & 0 \end{vmatrix}^2$, (Since $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$)

$= |a_3\mathbf{j} - a_2\mathbf{k}|^2 = a_3^2 + a_2^2$

Similarly, $|\mathbf{a} \times \mathbf{j}|^2 = a_1^2 + a_3^2$ and $|\mathbf{a} \times \mathbf{k}|^2 = a_1^2 + a_2^2$

Hence the required result can be given as

$2(a_1^2 + a_2^2 + a_3^2) = 2|\mathbf{a}|^2$.

103. B B; $\overrightarrow{PA} + \overrightarrow{PB} = (\overrightarrow{PA} + \overrightarrow{AC}) + (\overrightarrow{PB} + \overrightarrow{BC}) - (\overrightarrow{AC} + \overrightarrow{BC})$

$= \overrightarrow{PC} + \overrightarrow{PC} - (\overrightarrow{AC} - \overrightarrow{CB}) = 2\overrightarrow{PC} - 0$, $(\because \overrightarrow{AC} = \overrightarrow{CB})$

$\therefore \overrightarrow{PA} + \overrightarrow{PB} = 2\overrightarrow{PC}$.

104. B B; Required value $= \frac{\mathbf{b}.\mathbf{a}}{|\mathbf{b}|} \Big/ \frac{\mathbf{a}.\mathbf{b}}{|\mathbf{a}|} = \frac{|\mathbf{a}|}{|\mathbf{b}|} = \frac{7}{3}$.

105. A A; Required distance $= \dfrac{7-\frac{7}{2}}{\sqrt{1+4+9}} = \dfrac{\sqrt{7}}{2\sqrt{2}}$.

106. D D; Let AD be perpendicular and D be foot of perpendicular which divide BC in ratio $\lambda : 1$, then

$D\left(\frac{10\lambda-9}{\lambda+1}, \frac{4}{\lambda+1}, \frac{-\lambda+5}{\lambda+1}\right)$ .....(i)

The direction ratio of AD are $\frac{10\lambda-9}{\lambda+1}, \frac{4}{\lambda+1}, \frac{-\lambda+5}{\lambda+1}$ and direction ratio of BC are 19, – 4 and – 6.

Since $AD \perp BC$

$\Rightarrow 19\left(\frac{10\lambda-9}{\lambda+1}\right)-4\left(\frac{4}{\lambda+1}\right)-6\left(\frac{-\lambda+5}{\lambda+1}\right)=0$

$\Rightarrow \lambda=\frac{31}{28}$.

Hence on putting the value of λ in (i), we get required foot of the perpendicular *i.e.,* $\left(\frac{58}{59}, \frac{112}{59}, \frac{109}{59}\right)$.

Trick: The line passing through these points is $\frac{x+9}{19}=\frac{y-4}{-4}=\frac{z-5}{-6}$. Now co-ordinates of the foot lie on this line, so they must satisfy the given line. But here no point satisfies the line, hence answer is D;.

107. A

A; Equation of plane passing through the point (2, –1, –3) is,

Also, $A(x-2)+B(y+1)+C(z+3)=0$

Also, $3A+2B-4C=0$ and $2A-3B+2C=0$

$\therefore \frac{A}{-8}=\frac{B}{-14}=\frac{C}{-13}=k$, (Let)

So, $A=-8k, B=-14k, C=-13k$

Equation of required plane is,

$-k[8(x-2)+14(y+1)+13(z+3)]=0$

i.e., $8x+14y+13z+37=0$.

108. A

A; Direction cosines of line $=\left(\frac{2}{7}, \frac{3}{7}, \frac{-6}{7}\right)$

Now, $x'=1+\frac{2r}{7}, y'=-2+\frac{3r}{7}$ and $z'=3-\frac{6r}{7}$

$\therefore \left(1+\frac{2r}{7}\right)-\left(-2+\frac{3r}{7}\right)+\left(3-\frac{6r}{7}\right)=5 \Rightarrow r=1$.

109. D

110. C

C; We have, $\sin x+\sin^2 x=1$

or $\sin x=1-\sin^2 x$ or $\sin x=\cos^2 x$

$\therefore \cos^{12} x+3\cos^{10} x+3\cos^8 x+\cos^6 x-2$

$=\sin^6 x+3\sin^5 x+3\sin^4 x+\sin^3 x-2$

$=(\sin^2 x)^3+3(\sin^2 x)^2 \sin x$

$+3(\sin^2 x)(\sin x)^2+(\sin x)^3-2$

$=(\sin^2 x+\sin x)^3-2=(1)^3-2$

$[\because \sin x+\sin^2 x=1(\text{given})]$

$=-1$.

111. D

112. D

D; Let $S=1+2+3+\dots\dots\dots.+100$

$=\frac{100}{2}(1+100)=50(101)=5050$

Let $S_1=3+6+9+12+\dots\dots\dots+99$

$=3(1+2+3+4+\dots\dots\dots+33)$

$=3.\frac{33}{2}(1+33)=99\times17=1683$

Let $S_2=5+10+15+\dots\dots\dots+100$

$=5(1+2+3+\dots\dots\dots+20)$

$=5.\frac{20}{2}(1+20)=50\times21=1050$

Let $S_3 = 15 + 30 + 45 + ... + 90$

$= 15(1 + 2 + 3 + ... + 6)$

$= 15.\frac{6}{2}(1 + 6) = 45 \times 7 = 315$

$\therefore$ Required sum $= S - S_1 - S_2 + S_3$

$= 5050 - 1683 - 1050 + 315 = 2632.$

113. C

C; $\cot A, \cot B$ and $\cot C$ are in A. P.

$\Rightarrow \cot A + \cot C = 2\cot B \Rightarrow \frac{\cos A}{\sin A} + \frac{\cos C}{\sin C} = \frac{2\cos B}{\sin B}$

$\Rightarrow \frac{b^2 + c^2 - a^2}{2bc(ka)} + \frac{a^2 + b^2 - c^2}{2ab(kc)} = 2\frac{a^2 + c^2 - b^2}{2ac(kb)}$

$\Rightarrow a^2 + c^2 = 2b^2$. Hence a^2, b^2, c^2 are in A. P

Note : Students should remember this question as a fact.

114. D

D; Co-axial system $x^2 + y^2 + 2gx + c = 0$, (g variable)

L.H.S. $= \Sigma(g_2 - g_3)(h^2 + k^2 - c + 2g_1h) = 0$

Since $\Sigma(g_2 - g_3) = 0$ and $\Sigma g_1(g_2 - g_3) = 0$.

115. A

A; $7, 11$ have always to be in that group of three, therefore 3rd ticket may be chosen in 18 ways.

Hence required probability is $\frac{18}{^{20}C_3} = \frac{18.3.2}{20.19.18} = \frac{3}{190}$

116. B

B; The internal bisector of the angle A will divide the opposite side BC at D in the ratio of arms of the angle *i.e.* $AB = 3\sqrt{2}$ and $AC = 4\sqrt{2}$. Hence by ratio formula the point D is $\left(\frac{31}{7}, 1\right)$. Slope of AD by $\frac{y_2 - y_1}{x_2 - x_1} = 0$.

$\therefore$ Slope of a line perpendicular to AD is ∞.

Any line through C perpendicular to this bisector is $\frac{y - 5}{x - 5} = m = \infty$; $\therefore$ $x - 5 = 0$.

117. A

A; We have $\frac{1}{(x-1)(x^2+1)} = \frac{A}{(x-1)} + \frac{Bx + C}{(x^2+1)}$

$\Rightarrow 1 = A(x^2 + 1) + (Bx + C)(x - 1)$

If $x = 1$, then $A = \frac{1}{2}$ .....(i)

$A - C = 1 \Rightarrow C = -\frac{1}{2}$ .....(ii)

$A + B = 0 \Rightarrow B = -\frac{1}{2}$ .....(iii)

Putting these values, we get

$\frac{1}{(x-1)(x^2+1)} = \frac{1}{2}.\frac{1}{(x-1)} - \frac{x+1}{2(x^2+1)}$

Hence $\int \frac{1}{(x-1)(x^2+1)}dx = \frac{1}{2}\int \frac{dx}{(x-1)} - \frac{1}{2}\int \frac{x+1}{x^2+1}dx$

$= \frac{1}{2}\log(x-1) - \frac{1}{4}\log(x^2+1) - \frac{1}{2}\tan^{-1}x + c.$

118. D

D; Putting $x = \cot\theta$

$y = \cos^{-1}\left(\frac{x - x^{-1}}{x + x^{-1}}\right) = \cos^{-1}\left(\frac{x^2 - 1}{x^2 + 1}\right)$

$= \cos^{-1}(\cos 2\theta) = 2\theta \Rightarrow \frac{dy}{dx} = \frac{-2}{1 + x^2}$.

119. C

C; We can write

$aC_0 - (a + d)C_1 + (a + 2d)C_2 - ....$ upto $(n + 1)$ terms

$= a(C_0 - C_1 + C_2 - ....) + d(-C_1 + 2C_2 - 3C_3 + ...)$ ....(i)

Again, $(1 - x)^n = C_0 - C_1x + C_2x^2 - .... + (-1)^n C_n x^n$ ...(ii)

Differentiating with respect to x,

$-n(1-x)^{n-1} = -C_1 + 2C_2x - .... + (-1)^n C_n nx^{n-1}$ ....(iii)

Putting x =1 in (ii) and (iii), we get

$C_0 - C_1 + C_2 - .... + (-1)^n C_n = 0$

and $-C_1 + 2C_2 - .... + (-1)^n n.C_n = 0$

Thus the required sum to (n+1) terms, by (i)

=a.0 + d.0 = 0.

120. D 344