TRIGONOMETRY FORMULAS

$$\cos^2(x) + \sin^2(x) = 1 \qquad 1 + \tan^2(x) = \sec^2(x) \qquad \cot^2(x) + 1 = \csc^2(x)$$

$$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$$
$$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$$
$$\tan(x \pm y) = \frac{\tan(x) \pm \tan(y)}{1 \mp \tan(x)\tan(y)}$$

$$\sin(2x) = 2\sin(x)\cos(x)$$
$$\cos(2x) = \begin{cases} \cos^2(x) - \sin^2(x) \\ 2\cos^2(x) - 1 \\ 1 - 2\sin^2(x) \end{cases}$$
$$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$$

$$c^2 = a^2 + b^2 - 2ab\cos(C)$$
$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

$$\sin^2(x) = \frac{1 - \cos(2x)}{2}$$
$$\cos^2(x) = \frac{1 + \cos(2x)}{2}$$
$$\tan^2(x) = \frac{1 - \cos(2x)}{1 + \cos(2x)}$$

$$\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 + \cos(x)}{2}}$$
$$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 - \cos(x)}{2}}$$
$$\tan\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 - \cos(x)}{1 + \cos(x)}}$$

$$\sin(x)\sin(y) = \tfrac{1}{2}[\cos(x - y) - \cos(x + y)]$$
$$\cos(x)\cos(y) = \tfrac{1}{2}[\cos(x - y) + \cos(x + y)]$$
$$\sin(x)\cos(y) = \tfrac{1}{2}[\sin(x + y) + \sin(x - y)]$$
$$\cos(x)\sin(y) = \tfrac{1}{2}[\sin(x + y) - \sin(x - y)]$$

$$\sin(x) + \sin(y) = 2\sin\left(\frac{x + y}{2}\right)\cos\left(\frac{x - y}{2}\right)$$
$$\sin(x) - \sin(y) = 2\sin\left(\frac{x - y}{2}\right)\cos\left(\frac{x + y}{2}\right)$$
$$\cos(x) + \cos(y) = 2\cos\left(\frac{x + y}{2}\right)\cos\left(\frac{x - y}{2}\right)$$
$$\cos(x) - \cos(y) = -2\sin\left(\frac{x + y}{2}\right)\sin\left(\frac{x - y}{2}\right)$$

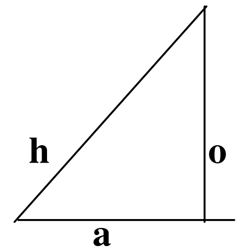
For two vectors **A** and **B, A·B** = ||**A**||||**B**||cos(θ)

The well known results: soh, cah, toa

soh: s stands for sine, o stands for opposite and h stands for hypotenuse, $\sin x = \dfrac{o}{h}$

cah: c stands for cosine, a stands for adjacent h stands for hypotenuse, $\cos x = \dfrac{a}{h}$

toa: t stands for tan, o stands for opposite and a stands for adjacent, $\tan x = \dfrac{o}{a}$



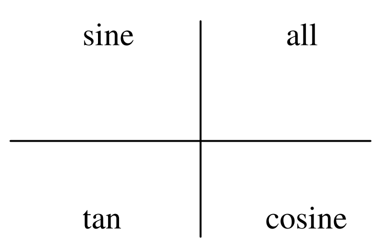
Where x is the angle between the hypotenuse and the adjacent.
Other three trigonometric functions have the following relations:

$\csc x = \dfrac{1}{\sin x} = \dfrac{h}{o}$, $\sec x = \dfrac{1}{\cos x} = \dfrac{h}{a}$ and $\cot x = \dfrac{1}{\tan x} = \dfrac{a}{o}$

Important values:

	0	$30^0 = \frac{\pi}{6}$	$45^0 = \frac{\pi}{4}$	$60^0 = \frac{\pi}{3}$	$90^0 = \frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined
csc	undefined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	undefined
cot	undefined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

$\sin(n\pi \pm x) = [\,?\,]\sin x$, $\cos(n\pi \pm x) = [\,?\,]\cos x$, $\tan(n\pi \pm x) = [\,?\,]\tan x$ **,** the sign **?** is for plus or minus depending on the position of the terminal side. One may remember the four-quadrant rule: (**A**ll **S**tudents **T**ake **C**alculus: A = all, S = sine, T = tan, C = cosine)



Example: Find the value of $\sin 300^0$. We may write $\sin 300^0 = \sin(2 \cdot 180^0 - 60^0) = [-]\sin 60^0 = -\frac{\sqrt{3}}{2}$, in this case the terminal side is in quadrant four where sine is negative.

In the following diagram, each point on the unit circle is labeled first with its coordinates (exact values), then with the angle in degrees, then with the angle in radians. Points in the lower hemisphere have both positive and negative angles marked.