MENSURATION

Mensuration is a branch of mathematics which deals with the measurements of lengths of lines, areas of surfaces and volumes of solids.

Mensuration may be divided into two parts.

1. Plane mensuration
2. Solid mensuration

Plane mensuration deals with perimeter, length of sides and areas of two dimensional figures and shapes.

Solid mensuration deals with areas and volumes of solid objects.

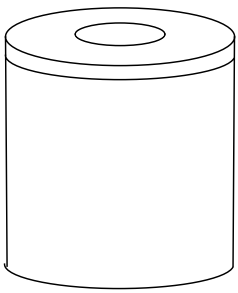
After learning this chapter you will be able to

* Recognise a cylinder, a cone and a sphere.
* Understand the properties of cylinder, cone and sphere.
* Distinguish between the structure of cylinder and cone.
* Derive the formula to find the surface area and volume of cylinder, cone and sphere.
* Solve simple problems pertaining to the surface area and volume of cylinder, cone and sphere.

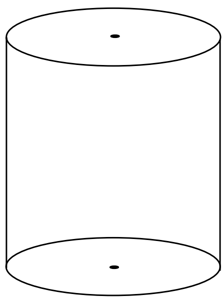
CYLINDER

Observe the following figures :

Road roller



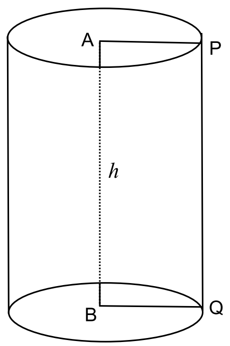
Circular based storage Tank



Cylinder

Wheels of a road roller, a circular based storage tank etc will suggest you, the concept of a right circular cylinder.

1. The right circular cylinder



A right circular cylinder is a solid described by revolution of a rectangle about one of its sides which remains fixed.

AP = Radius of the circular plane

AB = Axis of the cylinder

PQ = Height of the cylinder

Features of a right circular cylinder

1) A right circular cylinder has two plane surfaces, circular in shape.
2) The curved surface joining the plane surfaces is the lateral surface of the cylinder.
3) The two circular planes are parallel to each other and also congruent.
4) The line joining the centers of the circular planes is the axis of the cylinder.
5) All the points on the lateral surface of the right circular cylinder are equidistant from the axis.
6) Radius of circular plane is the radius of the cylinder.

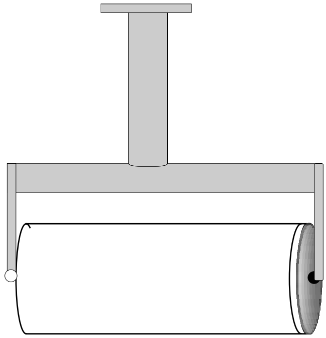
Two types of cylinders :

1. Hollow cylinder and
2. Solid cylinder

A hollow cylinder is formed by the lateral surface only.

Example : A pipe

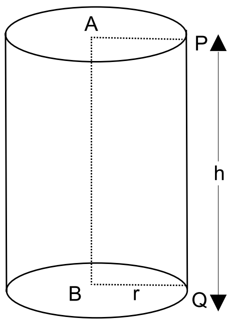
A solid cylinder is the region bounded by two circular plane surfaces and also the lateral surface.



Example : A garden roller

2. Surface area of a right circular cylinder

A. Lateral Surface area :



Activity :

1. Take a strip of paper having width equal to the height of the cylinder.
2. Wrap the strip around the lateral surface of the cylinder and cut the overlapping strip along the vertical line. (say PQ)
3. You will get a rectangular paper cutting which exactly covers the lateral surface.
4. Area of the rectangle is equal to the area of the curved surface of the cylinder.

Expression for the lateral surface area :

(i) Length of the rectangle is equal to the circumference, $l = 2\pi r$

(ii) Breadth of the rectangle b is the height of the cylinder = h

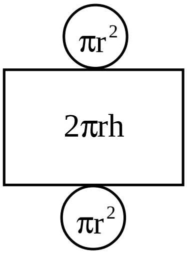
Area of the rectangle $A = l \times b$

Lateral surface area of the Cylinder $A = 2\pi r h$

$$\mathbf{A = 2\pi r h \text{ sq. units}}$$

> **Observe :**
>
> **Surface area of a thin hollow cylinder having circumference P and height is 'h' = Ph or = $2\pi r h$ ($\because P = 2\pi r$)**

B. Total surface area of a cylinder :



The total surface area of the cylinder = [Area of two circular bases] + [Lateral surface area of cylinder]

$= \pi r^2 + \pi r^2 + 2\pi rh$

$= 2\pi r^2 + 2\pi rh$

$= \boxed{2\pi r (r + h)}$ sq. units

> **Lateral surface area of a cylinder = $2\pi rh$ sq. units.**
>
> **Total surface area of a cylinder = $2\pi r (r+h)$ sq. units.**

> **Remember :**
>
> **Area is always expressed in square units**

Worked examples :

Example 1 **:** Find the lateral surface area of a cylinder whose circumference is 44 cm and height 10 cm.

Given : circumference = $2\pi r$ = 44 cms

and height = h = 10 cms

Solution : Lateral surface area of cylinder $= 2\pi rh$

$= 44 \times 10$

$= \boxed{440 \text{ sq. cm.}}$

Example 2 **:** Find the total surface area of the cylinder, given that the diameter is 10 cm and height is 12.5 cm.

Given : Diameter = d = 10 cms

$$\therefore \; r = \frac{d}{2} = 5 \text{ cms}$$

and height = h = 12.5 cms.

Solution : Total surface area of a cylinder = $2\pi r\ (r+h)$

$$= 2 \text{x} \frac{22}{7} \text{x} 5 \text{x} (5 + 12.5)$$

$$= 2 \text{x} \frac{22}{7} \text{x} 5 \text{x}\ 17.5$$

$$= \boxed{550 \text{ sq.cm.}}$$

Example 3 : The lateral surface area of a thin circular bottomed tin is 1760 sq. cm and radius is 10 cm. What is the height of the tin?

Given : The lateral surface area of a cylinder = $2\pi rh$ = 1760 sq. cm.

radius = r = 10 cms

Solution : Lateral surface area of a cylinder = $2\pi rh$

$$1760 = 2 \text{x} \frac{22}{7} \text{ x } 10 \text{ x } h$$

$$h = \frac{1760 \text{ x } 7}{2 \text{ x } 22 \text{ x } 10}$$

$$h = \boxed{28 \text{ cms}}$$

Exercise : 9.1

1) The radius of the circular base of a cylinder is 14 cm and height is 10 cm. Calculate the curved surface area of the cylinder.

2) The circumference of a thin hollow cylindrical pipe is 44 cm and length is 20 mts. Find the surface area of the pipe.

3) A cylinder has a diameter 20 cm and height 18 cm. Calculate the total surface area of the cylinder.

4) Lateral surface area of a cylinder is 1056 sq. cm and radius is 14 cm. Find the height of the cylinder.

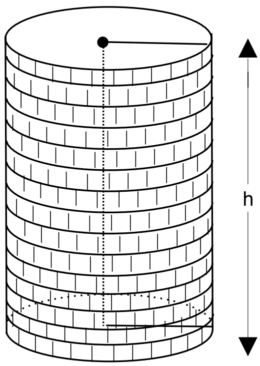
5) A mansion has twelve cylindrical pillars each having the circumference 50 cm and height 3.5 mts. Find the cost of painting the lateral surface of the pillars at Rs. 25 per squaremeter.

6) The diameter of a thin cylindrical vessel open at one end is 3.5 cm and height is 5 cm. Calculate the surface area of the vessel.

7) A closed cylindrical tank is made up of a sheet metal. The height of the tank is 1.3 meters and radius is 70 cm. How many square meters of sheet metal is required to make?

8) A roller having radius 35 cms and length 1 meter takes 200 complete revolutions to move once over a play ground. What is the area of the playground?

3. Volume of a right circular cylinder :



Activity :

1) A coin is placed on a horizontal plane.

2) Pile up the coins of same size one upon the other such that they form a right circular cylinder of height 'h'.

Volume of a cylinder = Bh

The area of the circular base B = πr^2 [B is the circular base of radius r]

Height of the cylinder = h

∴ Volume of the cylinder = $\pi r^2 h$ cubic units

Volume of the right circular cylinder of radius 'r' and height 'h' = $\pi r^2 h$ cubic units. Volume is expressed in cubic units.

Worked examples :

Example 1 : Find the volume of the cylinder whose radius is 7 cm and height is 12 cm.

Given : Radius of the cylinder = r = 7 cm

Height of the cylinder = h = 12 cm

Solution : Volume of the cylinder V = Bh

$$= \pi r^2 h$$

$$= \frac{22}{7} \times 7 \times 7 \times 12$$

$$= \frac{22}{7} \times 49 \times 12$$

= 1848 cubic cms.

∴ Volume of the cylinder = 1848 cc

Example 2 : Volume of the cylinder is 462 cc and its diameter is 7 cm. Find the height of the cylinder.

Given : Volume = V = 462 cc

Diameter = d = 7 cm ∴ r = 3.5 cm.

Solution : Volume of a cylinder V = $\pi r^2 h$

$$462 = \frac{22}{7} \times (3.5)^2 \times h$$

$$h = \frac{462 \times 7}{22 \times (3.5)^2}$$

h = 12 cm

∴ Height of the cylinder h = 12 cms

Exercise : 9.2

1) Area of the base of a right circular cylinder is 154 sq. cm and height is 10 cm. Calculate the volume of the cylinder.

2) Find the volume of the cylinder whose radius is 5 cm and height is 28 cm.

3) The circumference of the base of a cylinder is 88 cm and its height is 10 cm. Calculate the volume of the cylinder.

4) Volume of a cylinder is 3080 cc and height is 20 cm. Calculate the radius of the cylinder.

5) A cylindrical vessel of height 35 cm contains 11 litres of juice. Find the diameter of the vessel (one litre = 1000 cc.)

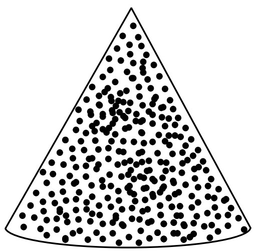
6) Volume of a cylinder is 4400 cc and diameter is 20 cm. Find the height of the cylinder.

7) The height of water level in a circular well is 7 mts and its diameter is 10 mts. Calculate the volume of water stored in the well.

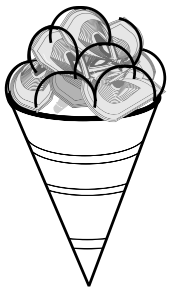
8) A thin cylindrical tin can hold only one litre of paint. What is the height of the tin if the diameter of the tin is 14 cm? (one litre = 1000 cc)

THE RIGHT CIRCULAR CONE

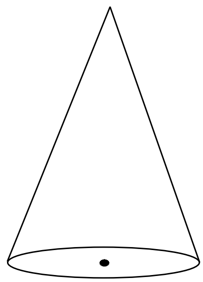
Observe the following figures :



Heap of sand



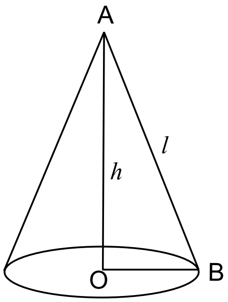
Ice cream cone



cone

A heap of sand, an Ice cream cone suggests you the concept of a right circular cone.

1. Surface area of a right circular cone :



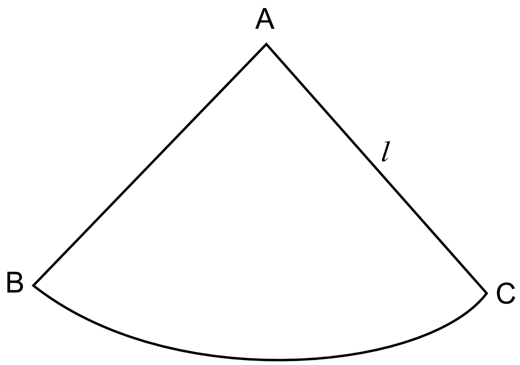
A right circular cone is a solid generated by the revolution of a right angle triangle about one of the sides containing the right angle.

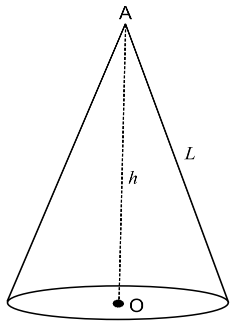
Properties of a right circular cone :

1) A cone has a circular plane as its base.
2) The point of intersection of the axis of the cone and slant height is the vertex of the cone (A).
3) The curved surface which connects the vertex and circular edge of the base is the lateral surface of the cone.
4) The line joining the vertex and the center of the circular base is the height of the cone (AO = h)

> **Remember :**
>
> The distance between the vertex and any point on the circumference of the base is the slant height.

A. The curved surface area :



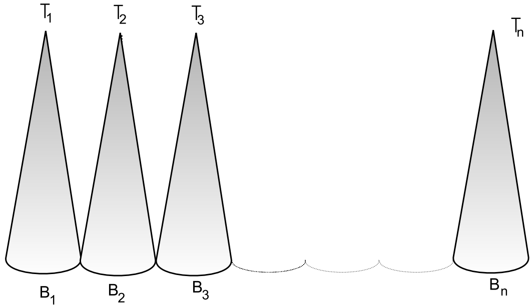


Activity :

1) Take a right circular cone.
2) Wrap the curved surface with a piece of paper.
3) Cut the paper along the length of slant height say AB
4) Take out the paper which exactly covers the curved surface.
5) Spread the paper on a plane surface.

> **Observe :**
>
> Radius of the circular section is equal to slant height of the cone = l

This circular section can be divided into small triangles as shown in the figure say T_1, T_2, T_3 ... T_n



From the figure lateral surface area of the circular section = sum of the areas of each triangle.

$$= T_1 + T_2 + \ldots\ldots T_n$$

Area of the Triangle = $\frac{1}{2}$ x base x height

i.e. Area of $T_1 = \frac{1}{2} \times B_1 \times l$

$$T_2 = \frac{1}{2} \times B_2 \times l$$

$$T_3 = \frac{1}{2} \times B_3 \times l$$

Area of $T_n = \frac{1}{2} \times B_n \times l$

Area of circular section $= \frac{1}{2} B_1 l + \frac{1}{2} B_2 l + \ldots\ldots\ldots\ldots\ldots + \frac{1}{2} B_n l$

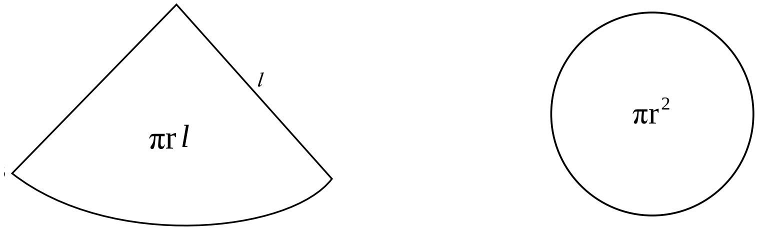
$$= \frac{1}{2} l \, (B_1 + B_2 + \ldots\ldots + B_n)$$

$$= \frac{1}{2} l \, (2\pi r) \qquad [B_1 + B_2 + \ldots. + B_n = 2\pi r]$$

$$= \frac{1}{2} l \times 2\pi r$$

∴ Area of the curved surface of a cone = $\pi r l$ sq. units

B. Total Surface area of a cone :



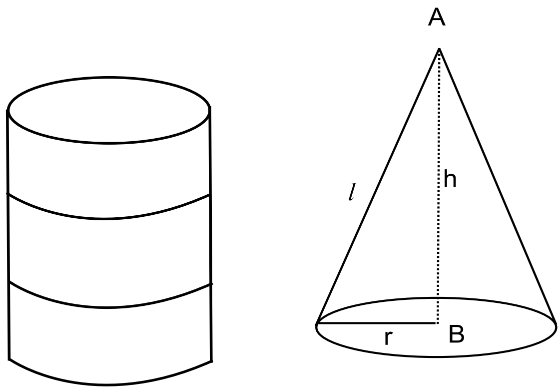
Total surface area of a cone = Area of circular base + Area of the curved surface

$$= \pi r^2 + \pi r l$$

$$= \pi r (r + l)$$

Total surface area of a cone = $\pi r [r + l]$ sq. units

2. Volume of a right circular cone



Suggested Activity :

1) Take a conical cup and a cylindrical vessel of the same radius and height.
2) Fill the conical cup with water up to its brim.
3) Pour the water into cylindrical vessel.
4) Count how many cups of water is required to fill the cylindrical vessel upto its brim.

Observe that exactly three cups of water is required to fill the vessel.

Volume of a cylinder = 3 x volume of a cone having same base and height.

$\therefore$ Volume of a cone $= \frac{1}{3}$ of the volume of a cylinder having same base and height.

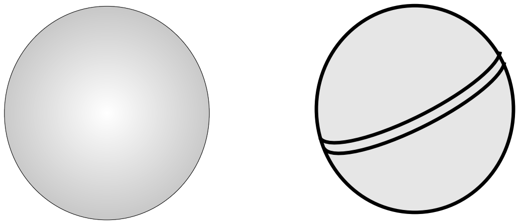
$= \frac{1}{3} \times Bh$ [$\because$ Volume of a cylinder = Bh]

$= \frac{1}{3}\pi r^2 h$ [$\because$ $B = \pi r^2$]

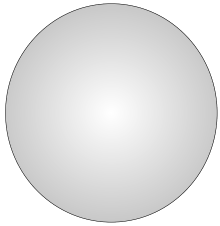
Volume of a cone of radius r and height h = $\frac{1}{3}\pi r^2 h$ cubic units.

THE SPHERE

Observe the following figures



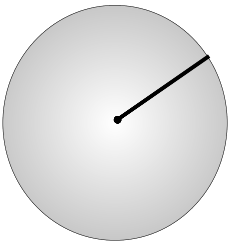
Shot put Ball



Sphere

A shotput, a ball etc, will suggest you the concept of a sphere.

Properties of a sphere :



A sphere is a solid described by the revolution of a semi circle about a fixed diameter.

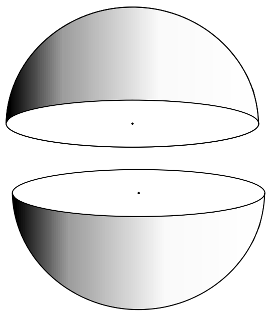
1) A sphere has a centre
2) All the points on the surface of the sphere are equidistant from the centre.
3) The distance between the centre and any point on the surface of the sphere is the radius of the sphere.

Remember :

A plane through the centre of the sphere divides it into two equal parts each called a hemisphere.

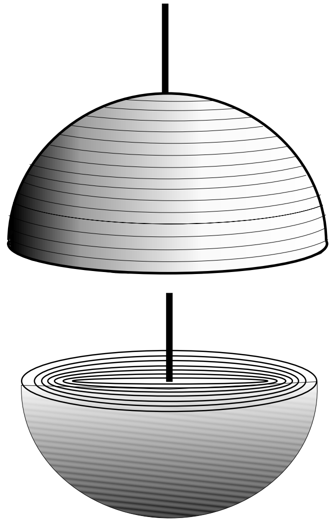
1. Surface Area of a sphere :

Activity :



1) Consider a sphere of radius r.
2) Cut the solid sphere into two equal halves.
3) Fix a pin at the top most point of a hemisphere.
4) Starting from the centre point of the curved

surface of the hemisphere, wind a uniform thread so as to cover the whole curved surface of the hemisphere.



5) Measure the length of the thread.

6) Similarily, fix a pin at the center of the plane circular surface.

7) Starting from the centre, wind the thread of same thickness to cover the whole circular surface.

8) Unwind and measure the lengths of the threads.

9) Compare the lengths.

What do you observe from both the activities?

It is found that the length of the thread required to cover the curved surface is twice the length required to cover the circular plane surface.

Area of the plane circular surface $= \pi r^2$

$\therefore$ Curved surface area of a hemisphere $= 2\pi r^2$

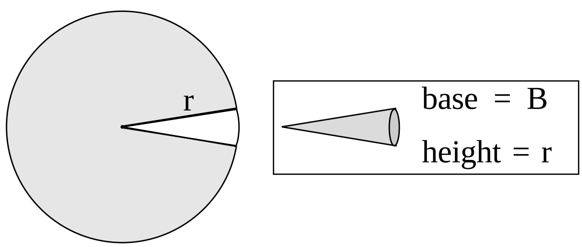
Surface area of the whole sphere $= 2\pi r^2 + 2\pi r^2$

$= 4\pi r^2$

Surface area of a sphere of radius r = $4\pi r^2$ sq. units.

2. Volume of a sphere

Observe the following figures.



A solid sphere is made up of miniature cones whose height is equal to the radius of the sphere and each having circular base.

Volume of each cone $= \frac{1}{3}$ x base x height

Volume of cone 1 $= \frac{1}{3} \times B_1 \times r$

Volume of cone 2 $= \frac{1}{3} \times B_2 \times r$ etc

Volume of cone n $= \frac{1}{3} \times B_n \times r$

Volume of the sphere = Sum of the volumes of all the cones

$$= \frac{1}{3} \times B_1 \times r + \frac{1}{3} \times B_2 \times r + ... \frac{1}{3} \times B_n \times r$$

$$= \frac{1}{3} r (B_1 + B_2 + ... + B_n) \quad [\because \text{T.S.A. of sphere } 4\pi r^2]$$

$$= \frac{1}{3} r (B) \quad [\because \text{Surface area of sphere}]$$

$$= \frac{1}{3} r \times 4\pi r^2$$

$$= \frac{4}{3}\pi r^3$$

Volume of the sphere $= \frac{4}{3}\pi r^3$ cubic units

Volume of the hemisphere $= \frac{1}{2} \times \frac{4}{3}\pi r^3$

$$= \frac{2}{3}\pi r^3$$

$\therefore$ Volume of a hemisphere $= \frac{2}{3}\pi r^3$ cubic untis

Remember at a glance :

Solid	Curved Surface area	Total Surface area	Volume
Cylinder	$2\pi rl$	$2\pi r\ (r+1)$	$\pi r^2 h$
Cone	πrl	$\pi r\ (r+1)$	$\frac{1}{3}\pi r^2 h$
Sphere	$4\pi r^2$	$4\pi r^2$	$\frac{4}{3}\pi r^3$
Solid hemisphere	$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$

SCALE DRAWING

Area of triangle	**= $\frac{1}{2}$ x base x height**	**= $\frac{1}{2}$ bh**
Area of rectangle	**= length x breadth**	**= *l*b**
Area of trapezium	**= $\frac{1}{2}$ x height x (sum of two parallel sides)**	**= $\frac{1}{2}$ h (a+b)**

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B