

## Algebra, Algebra of Expressions

- 
1.  $(a + b)^2 = a^2 + 2ab + b^2$ ;  $a^2 + b^2 = (a + b)^2 - 2ab$
  2.  $(a - b)^2 = a^2 - 2ab + b^2$ ;  $a^2 + b^2 = (a - b)^2 + 2ab$
  3.  $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
  4.  $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$ ;  $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$
  5.  $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$ ;  $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$
  6.  $a^2 - b^2 = (a + b)(a - b)$
  7.  $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
  8.  $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
  9.  $a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \dots + b^{n-1})$
  10.  $a^n = a.a.a \dots n$  times
  11.  $a^m.a^n = a^{m+n}$
  12.  $\frac{a^m}{a^n} = a^{m-n}$  if  $m > n$   
 $= 1$  if  $m = n$   
 $= \frac{1}{a^{n-m}}$  if  $m < n$ ;  $a \in R, a \neq 0$
  13.  $(a^m)^n = a^{mn} = (a^n)^m$
  14.  $(ab)^n = a^n.b^n$
  15.  $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
  16.  $a^0 = 1$  where  $a \in R, a \neq 0$
  17.  $a^{-n} = \frac{1}{a^n}, a^n = \frac{1}{a^{-n}}$
  18.  $a^{p/q} = \sqrt[q]{a^p}$
  19. If  $a^m = a^n$  and  $a \neq \pm 1, a \neq 0$  then  $m = n$
  20. If  $a^n = b^n$  where  $n \neq 0$ , then  $a = \pm b$
  21. If  $\sqrt{x}, \sqrt{y}$  are quadratic surds and if  $a + \sqrt{x} = \sqrt{y}$ , then  $a = 0$  and  $x = y$
  22. If  $\sqrt{x}, \sqrt{y}$  are quadratic surds and if  $a + \sqrt{x} = b + \sqrt{y}$  then  $a = b$  and  $x = y$
  23. If  $a, m, n$  are positive real numbers and  $a \neq 1$ , then  $\log_a mn = \log_a m + \log_a n$
  24. If  $a, m, n$  are positive real numbers,  $a \neq 1$ , then  $\log_a \left(\frac{m}{n}\right) = \log_a m - \log_a n$
  25. If  $a$  and  $m$  are positive real numbers,  $a \neq 1$  then  $\log_a m^n = n \log_a m$
  26. If  $a, b$  and  $k$  are positive real numbers,  $b \neq 1, k \neq 1$ , then  $\log_b a = \frac{\log_k a}{\log_k b}$
  27.  $\log_b a = \frac{1}{\log_a b}$  where  $a, b$  are positive real numbers,  $a \neq 1, b \neq 1$
  28. if  $a, m, n$  are positive real numbers,  $a \neq 1$  and if  $\log_a m = \log_a n$ , then  $m = n$

29. if  $a + ib = 0$  where  $i = \sqrt{-1}$ , then  $a = b = 0$

30. if  $a + ib = x + iy$ , where  $i = \sqrt{-1}$ , then  $a = x$  and  $b = y$

31. The roots of the quadratic equation  $ax^2 + bx + c = 0$ ;  $a \neq 0$  are  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The solution set of the equation is  $\left\{ \frac{-b + \sqrt{\Delta}}{2a}, \frac{-b - \sqrt{\Delta}}{2a} \right\}$

where  $\Delta = \text{discriminant} = b^2 - 4ac$

32. The roots are real and distinct if  $\Delta > 0$ .

33. The roots are real and coincident if  $\Delta = 0$ .

34. The roots are non-real if  $\Delta < 0$ .

35. If  $\alpha$  and  $\beta$  are the roots of the equation  $ax^2 + bx + c = 0$ ,  $a \neq 0$  then

$$\text{i) } \alpha + \beta = \frac{-b}{a} = -\frac{\text{coeff. of } x}{\text{coeff. of } x^2}$$

$$\text{ii) } \alpha \cdot \beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coeff. of } x^2}$$

36. The quadratic equation whose roots are  $\alpha$  and  $\beta$  is  $(x - \alpha)(x - \beta) = 0$

$$\text{i.e. } x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

i.e.  $x^2 - Sx + P = 0$  where  $S = \text{Sum of the roots}$  and  $P = \text{Product of the roots}$ .

37. For an arithmetic progression (A.P.) whose first term is  $(a)$  and the common difference is  $(d)$ .

$$\text{i) } n^{\text{th}} \text{ term} = t_n = a + (n - 1)d$$

$$\text{ii) } \text{The sum of the first } (n) \text{ terms} = S_n = \frac{n}{2}(a + l) = \frac{n}{2}\{2a + (n - 1)d\}$$

where  $l = \text{last term} = a + (n - 1)d$ .

38. For a geometric progression (G.P.) whose first term is  $(a)$  and common ratio is  $(\gamma)$ ,

$$\text{i) } n^{\text{th}} \text{ term} = t_n = a\gamma^{n-1}.$$

ii) The sum of the first  $(n)$  terms:

$$\begin{aligned} S_n &= \frac{a(1 - \gamma^n)}{1 - \gamma} && \text{if } \gamma < 1 \\ &= \frac{a(\gamma^n - 1)}{\gamma - 1} && \text{if } \gamma > 1 \\ &= na && \text{if } \gamma = 1 \end{aligned}$$

39. For any sequence  $\{t_n\}$ ,  $S_n - S_{n-1} = t_n$  where  $S_n = \text{Sum of the first } (n) \text{ terms}$ .

$$40. \sum_{\gamma=1}^n \gamma = 1 + 2 + 3 + \dots + n = \frac{n}{2}(n + 1).$$

$$41. \sum_{\gamma=1}^n \gamma^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n}{6}(n + 1)(2n + 1).$$

$$42. \sum_{\gamma=1}^n \gamma^3 = 1^3 + 2^3 + 3^3 + 4^3 + \cdots + n^3 = \frac{n^2}{4}(n+1)^2.$$

$$43. n! = (1).(2).(3).\dots.(n-1).n.$$

$$44. n! = n(n-1)! = n(n-1)(n-2)! = \dots$$

$$45. 0! = 1.$$

$$46. (a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3 + \cdots + b^n, n > 1.$$